

Why the Tie Effect Occur?: Structural Interference Approach to Single-Digit Addition and Multiplication

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What is Tie Effect?

Tie effect is the phenomenon that tie problems like $3+3$ are solved easily than non-tie problems like $3+4$ in production task.

Table 1: Performances of tie and non-tie problems.

Problem type	Tie	Non-tie
Example	$3 + 3, 6 \times 6$	$3 + 4, 5 \times 8$
Latency	faster	slower
Accuracy	more	less

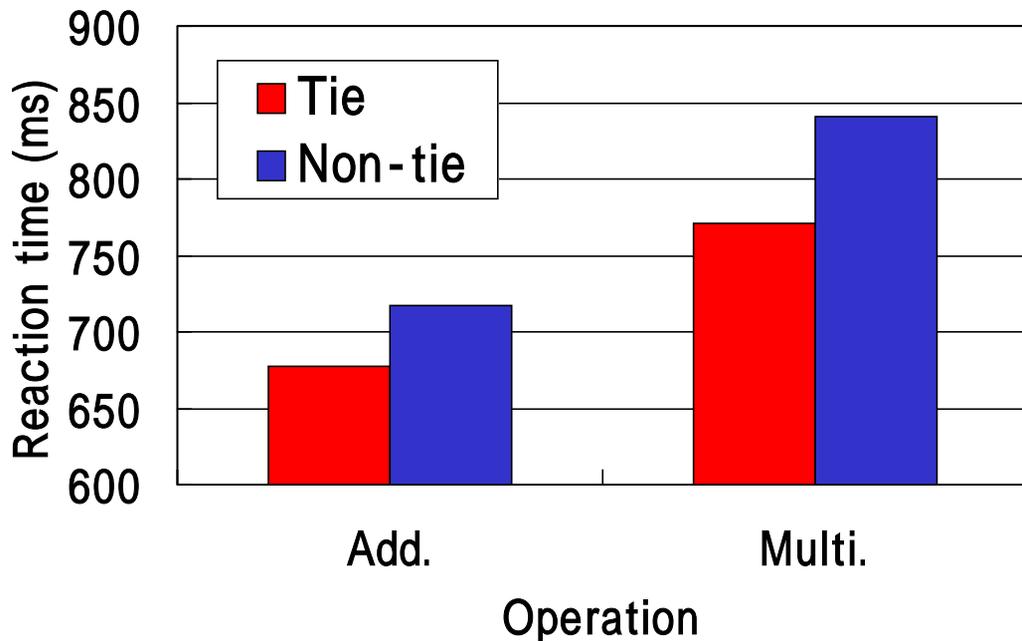


Figure 1: Tie effect. Reaction times as a function of operation (addition/multiplication) and tie/non-tie.

(Data from Shimada, 2000)

Model of Arithmetic Fact Retrieval

Performance is a function of two factors:

- (1) The strength of **correct associations**: The strength of association between problem and answer.
- (2) The strength of **interference associations**: The strength of interference of candidates, or undesirable associations.

(e.g. Campbell, 1995; Siegler, 1988)

Table 2: Example of two factors

Problem	Correct Association	Interference Association
$3 + 4$	$3 + 4$ 7	$3 + 4$ 5,6,8,9,..etc.
4×7	4×7 28	4×7 21,24,32,35...etc.

Purpose of This Study

The purpose of this study was to investigate **which factors caused tie effect.**

Outline of This Study

Experimental study, which evaluated the strength of associations between problem and answer, or correct associations for tie and non-tie problems.

Theory and its simulation, which is named structural interference model.

Method

The strength of correct associations were evaluated utilizing **number matching task** (LeFevre, et al., 1988).

Design

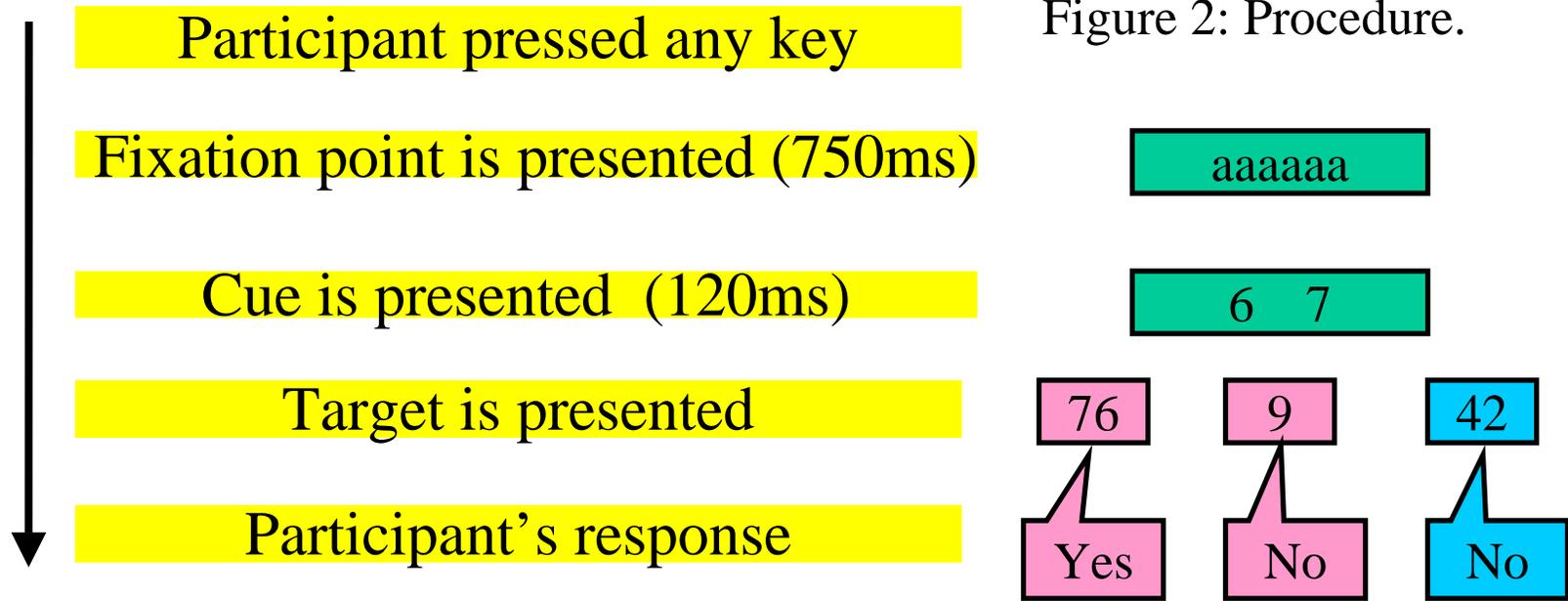
Operation (addition and multiplication) and problem type (tie and non-tie) were manipulated in this experiment.

Participants

Eight university students participated in this experiment.

Procedure (Number Matching Task)

Figure 2: Procedure.



Participants were required to press “yes” key when the target was either x or y , $10x+y$, $10y+x$ if the cue were x and y , and “no” key when either unit or decade included in the target differed from the cue numbers, **even though the target was sum or product of the cue.**

Utilizing number matching task, when answer target (sum or product) is presented, latency until pressing “no” key for some problem types which have **strong associations** between problem and answer are **slower** than problem types which have weak associations because of **unintentional activation of sum or product.**

Results

RTs of trials which have answer targets (sum or product) were analyzed.

(1) Tie conditions had long reaction times both operations.

The strength of associations for tie problems are stronger than for non-tie problems both operations.

(2) The difference of reaction times between tie and non-tie condition in addition was greater than in multiplication.

The difference of the strength of associations in addition is greater than in multiplication.

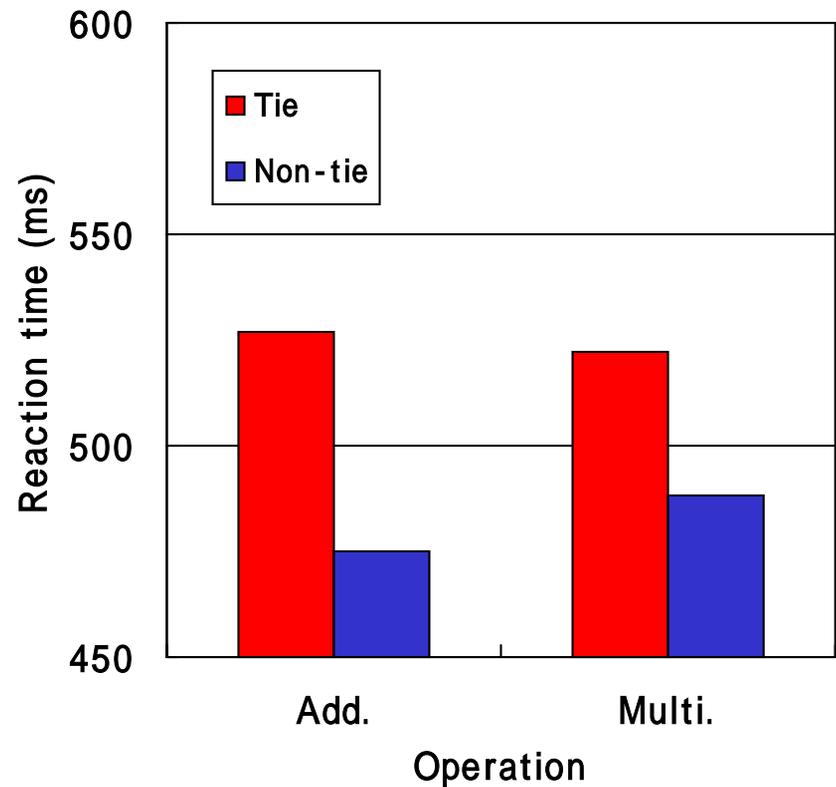


Figure 3: Reaction times as a function of operation (addition/multiplication) and problem type(tie/non-tie). As a result of ANOVA, interaction was significant ($p < .05$).

Structural Interference Model

Structural Interference Model, which consists of two stages:

Learning of associations and **processing**, can explain some phenomena, for example, problem size effect, the difference of RT between addition and multiplication, and tie effect.

The outline of structural interference model is following below.

Learning of associations

The strength of associations for each problems is determined by following formula.

$$\frac{dAss(a,b,r,c)}{dt} = a_p (1 - Ass) + a_o (1 - Ass) + a_f (0 - Ass) + a_s (0 - Ass)$$

Where t is time, or frequency, (a,b,r,c) stands for arithmetic facts like (3,4,+,7), Ass(a,b,r,c), which is a function of t and (a,b,r,c), is the strength of association, and four a*s are constant increment, operand-base increment, decrement by fan effect, and decrement by magnitude similarity, respectively.

Following formula can be given with $Ass(t=0)=0$ for any facts (a,b,r,c), and t .

$$Ass(a,b,r,c) = \frac{a_p + a_o}{a_p + a_o + a_f + a_s}$$

Performance of production

Performance of production for each problems are determined by following formula.

$$P(a,b,r,c) = \frac{I(a,b,r,c)a_s}{Ass(a,b,r,c)}$$

Where $P(a,b,r,c)$ is performance for “arb=?”: The smaller P, the faster and more accurately fact is retrieved. a_s is magnitude similarity, and $I(a,b,r,c)$, which is determined by sum of “Ass”s of similar facts for each (a,b,r,c) (e.g. table-related fact, c.f. Campbell, 1995) is the strength of interference to “arb=?”.

Results of Simulation

Learning of association

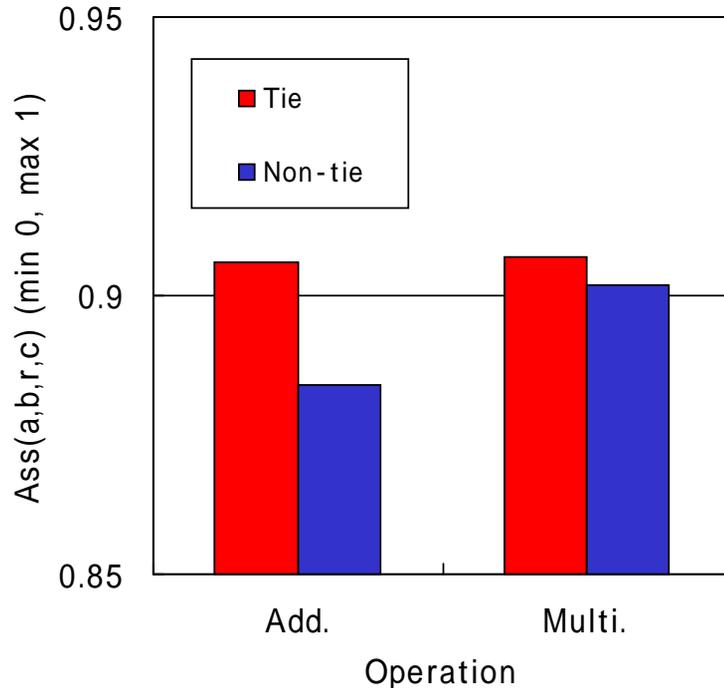


Figure 4: Values of $Ass(a,b,r,c)$.

Performance of production

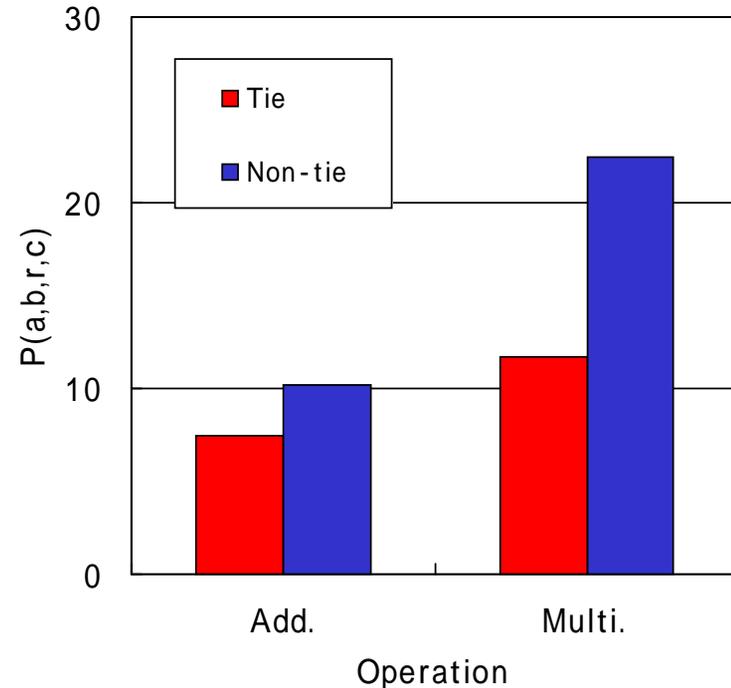


Figure 5: Values of $P(a,b,r,c)$.

These patterns of $Ass(a,b,r,c)$ and $P(a,b,r,c)$ can explain patterns of correct association (Figure 2) and performance of production task (Figure 1) in each conditions, respectively.

Conclusion: Why does tie effect occur?

Main factor which cause tie effect is **different for each operation.**

Addition

Tie effect occur **because correct associations** of tie are stronger than of non-tie.

Correct association: Tie correct associations (red) are stronger than non-tie. (That was showed in present experiment).

Interference: The number of table-related numbers (blue) of tie are approximately as many as of non-tie, so interference to tie are approximately as strong as to non-tie.

Table 2 : Addition table.

	2	3	4	5	6	7	8	9
2	4	5	6	7	8	9	10	11
3	5	6	7	8	9	10	11	12
4	6	7	8	9	10	11	12	13
5	7	8	9	10	11	12	13	14
6	8	9	10	11	12	13	14	15
7	9	10	11	12	13	14	15	16
8	10	11	12	13	14	15	16	17
9	11	12	13	14	15	16	17	18

Table 3 : Multiplication table.

	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	<u>16</u>	20	24	28	32	36
5	10	<u>15</u>	20	<u>25</u>	30	35	40	45
6	12	18	<u>24</u>	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Multiplication

Tie effect occur **mainly because interferences** to tie are stronger than to non-tie. But correct associations also contribute to tie effect in a little.

Correct association: Tie correct associations are stronger than non-tie, but the difference of the strength of correct associations (red) between tie and non-tie are smaller than in addition (That was showed in present experiment) .

Interference: Tie multiplication facts have less table-related numbers (blue) than non-tie, so the strength of interference to tie is weaker than to non-tie. (see Campbell & Graham, 1985, Graham & Campbell, 1992)

Table 4: Conclusion. Which factor contributes to tie effect in addition and multiplication?

	Addition	Multiplication
Correct Association		
Interference	×	

Note: is main factor. factor account for tie effect in a little. × factor does not contribute to tie effect.

References

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